Beware of the DAG!

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"Causal Discovery"

- Gather observational data on system
- Infer *conditional independence properties* of joint distribution
- Fit a *DIRECTED ACYCLIC GRAPH* model to represent these
- Interpret this model *CAUSALLY*

OK???

- When is this meaningful?
- What does it mean?
- When is it trustworthy?
- How can it be formalised?
- What assumptions are required?
- How can they be justified?

Seeing and Doing

- Causality is about the effects of *interventions*
- To discover these we really should *experiment*
- If we can't, is there anything sensible we can conclude from observational data?
- No amount of clever analysis of purely observational data can replace experimentation

SEEING

- Association
  - describe stochastic dependence and independence
- Conditional Independence

$$X \perp\!\!\!\perp Y \mid Z \; [P]$$

  - attribute of joint probability distribution P

$$p(x, y \mid z) = p(x \mid z)\, p(y \mid z)$$
$$p(x \mid y, z) = p(x \mid z)$$

Properties of CI

$$X \perp\!\!\!\perp Y \mid Z \Rightarrow Y \perp\!\!\!\perp X \mid Z$$
$$X \perp\!\!\!\perp Y \mid X$$
$$X \perp\!\!\!\perp Y \mid Z, \quad W \preceq Y \Rightarrow X \perp\!\!\!\perp W \mid Z$$
$$X \perp\!\!\!\perp Y \mid Z, \quad W \preceq Y \Rightarrow X \perp\!\!\!\perp Y \mid (W, Z)$$
$$\left.\begin{array}{l} X \perp\!\!\!\perp Y \mid Z \\ \text{and} \\ X \perp\!\!\!\perp W \mid (Y, Z) \end{array}\right\} \Rightarrow X \perp\!\!\!\perp (Y, W) \mid Z$$

Algebraic Representation

- We can make these properties the *axioms* of a formal algebraic theory
  - *separoid*
  - *semi-graphoid*
- Other applications too
- Can use to represent and manipulate CI without referring back to *P*
- Not complete

Graphical Representation

- Certain collections of CI properties can be described and manipulated using a DAG representation
  – *very far from complete*
- Each CI property *is represented by* a graphical separation property
  – *d-separation*
  – *moralization*

Moralization: 1

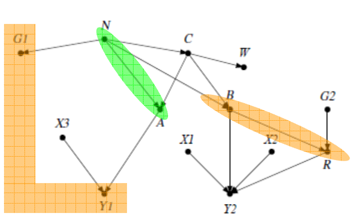


Figure 6.1: Directed graph $\mathcal{D}$ for criminal evidence

$(B, R) \perp\!\!\!\perp (G1, Y1) \mid (A, N)$??

Moralization: 2

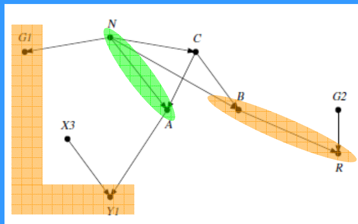


Figure 6.2: Ancestral subgraph $\mathcal{D}'$

$(B, R) \perp\!\!\!\perp (G1, Y1) \mid (A, N)$??

Moralization: 3

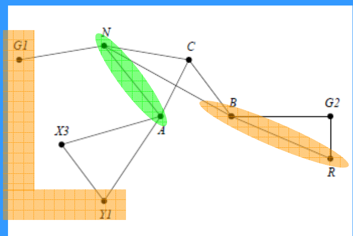


Figure 6.3: Moralized ancestral subgraph $\mathcal{G}'$

$(B, R) \perp\!\!\!\perp (G1, Y1) \mid (A, N)$??

DAG construction

Given a distribution over ordered set of variables

$$V^N = (V_1, \ldots, V_N),$$

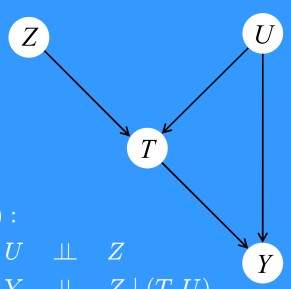
Construct DAG with the (V_i) as vertices as follows:

For $i = 0, \ldots, N-1$:

- S_i = subset of V^i such that $C_{i+1} : V_{i+1} \perp\!\!\!\perp V^i \mid S_i$
- Insert an arrow from each $V_j \in S_i$ into V_{i+1}

Resulting DAG represents exactly those CI properties algebraically deducible from $C_1, \ldots, C_N$

Example



(Z, U, T, Y) :

$U \perp\!\!\!\perp Z$

$Y \perp\!\!\!\perp Z \mid (T, U)$

Points to Remember

- The graph is *nothing but* an indirect way of describing a set of CI relationships
- Clear semantics (d-separation)
- May be several representations, or none
- Arrows have no intrinsic meaning
  - CI is non-directional!
- Represented relationships unaffected by others unmentioned, omitted variables,…
- Nothing to do with causality…

"Reification"

In an *associational* DAG:

- (Some) arrows represent direction of influence, direct cause,…
- (Some) directed paths represent "causal pathways"
- If these exist in all equivalent DAG representations they are "truly causal"

What do above causal terms mean?

Why/how do they relate to DAGs?

Probabilistic Causality

- *Weak Causal Markov* assumption:
  - If *X* and *Y* have no common cause (including each other), they are probabilistically independent
- *Causal Markov* assumption:
  - A variable is probabilistically independent of its non-effects, given its direct causes

What do above causal terms mean?

When/how widely do these assumptions hold?

Causal DAG

A *causal DAG* is a DAG in which:

1) the lack of an arrow from V_j to V_m can be interpreted as the absence of a direct causal effect of V_j on V_m (relative to the other variables on the graph)

2) all common causes (even if unmeasured) of any pair of variables on the graph are themselves on the graph

Then Causal Markov $\Rightarrow$ Markov

Converse???

Some problems

- Multiple interpretations of the same object (DAG)
  - – ambiguous and confusing
- Causal interpretation informal and obscure

➢ We need a clear formal language, with explicit semantics, by which we can describe and manipulate causal properties

➢ This should not commit us to any particular causal assumptions

Causality and Intervention

- Causality = response of a system to an (actual or proposed) intervention
- Typically we can only observe undisturbed ("idle") system
- Causal inference will requires *assumptions* relating idle and interventional regimes
- Want a language to express such assumptions
- *Quite separately*, want guidance on the appropriateness of assumptions in specific contexts

Intervention Variables

Variable F_X describing *kind* of intervention at X

$F_X = x$: *manipulate* X to value x
$F_X = \emptyset$: hands off!

Different settings of intervention variables determine different joint distributions (so parameter, not random, variables)

Assume $F_X = x \Rightarrow X = x$ (can relax…)

— no other hard-and-fast assumptions

A *Possible* Assumption: Modularity

- Knowing value a taken by A, do not need to know HOW this happened (i.e. by intervention or naturally) in order to predict B
- Conditional distribution of B given A is a modular component, transferable across regimes
- $p(B \mid A, F_A)$ does not depend on F_A
- $B \perp\!\!\!\perp F_A \mid A$

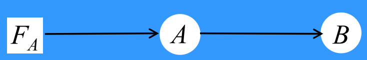
Extended Conditional Independence

- Such "extended CI" properties can be formally manipulated using the same algebraic rules as for regular CI
  - (must not have an intervention as first term in a CI)
- Allows us to determine consequences of our input assumptions

➢ *Causal inference*

Augmented DAG

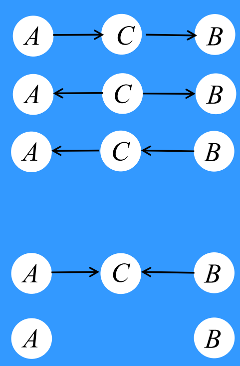
- Include intervention indicators in DAG



$$B \perp\!\!\!\perp F_A \mid A$$

- Explicit causal interpretation
  - using *d-separation* to express ECI
  - causality *NOT* (directly) represented by arrows

Markov Equivalence



$A \perp\!\!\!\perp B \mid C$

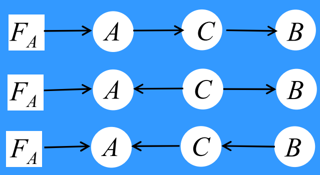
$A \perp\!\!\!\perp B \mid C$

$A \perp\!\!\!\perp B \mid C$

$A \perp\!\!\!\perp B$

$A \perp\!\!\!\perp B$

Markov Non-Equivalence

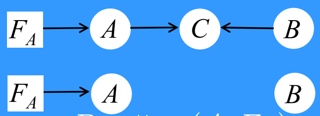


$$\begin{cases} C \perp\!\!\!\perp F_A & \mid A \\ B \perp\!\!\!\perp (A, F_A) & \mid C \end{cases}$$

$$\begin{cases} C \perp\!\!\!\perp F_A & \\ B \perp\!\!\!\perp (A, F_A) & \mid C \end{cases}$$

$$\begin{cases} (B, C) \perp\!\!\!\perp F_A & \\ B \perp\!\!\!\perp A & \mid (F_A, C) \end{cases}$$

$$B \perp\!\!\!\perp A \mid (F_A, C)$$

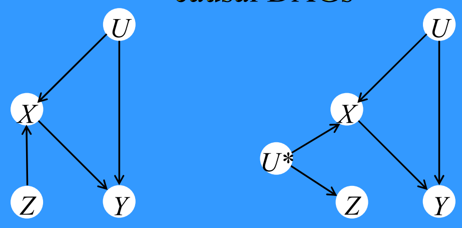


$$\begin{cases} B \perp\!\!\!\perp (A, F_A) & \\ C \perp\!\!\!\perp F_A & \mid (A, B) \end{cases}$$

$$\{ B \perp\!\!\!\perp (A, F_A)$$

$$B \perp\!\!\!\perp (A, F_A)$$

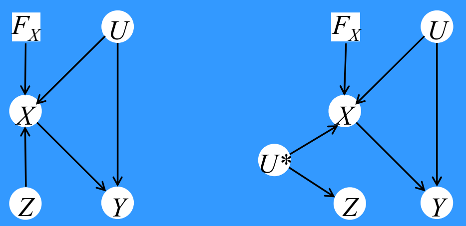
Instrumental Variable — causal DAGs



$$U \perp\!\!\!\perp Z$$

$$Y \perp\!\!\!\perp Z \mid (U, X)$$

Instrumental Variable



$$\perp\!\!\!\perp \{U, Z, F_X\}$$

$$Y \perp\!\!\!\perp (Z, F_X) \mid (U, X)$$

Intervention DAGs

- Pearl's interpretation of a DAG as causal
  - implicit addition of an intervention node for each random node
- Relates regimes that intervene on any set of variables (or none)
- When valid, allows causal inference from observational data

Can we just add intervention variables to a DAG?

- Behaviour of system when kicked need not bear any relationship to its behaviour when observed
- If $A \perp\!\!\!\perp B \quad (A \perp\!\!\!\perp B \mid \text{ancestors})$, on adding interventions, neither of A nor B can cause the other (converse of weak causal Markov property??)
  - why need this be?

The way ahead?

A. Use contextual understanding of problem to justify causal input assumptions
   - randomization
   - Mendelian randomization
   - natural experiments

B. Perform various real experiments
   - Hunt for ECI properties
   - Apply "causal discovery" to construct augmented DAG

Thank you!