The effect of treatment on the treated: a decision theoretic perspective

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Outline

- Motivation
- Counterfactual definition
- Problems
- Intro to Decision theoretic approach (DT)
- DT definition
- Identifiability
- Intrumental variables
- Conclusions
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<th>Econometrics</th>
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<td>- A programme for adult learning is opened in a community</td>
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<td>- How do you <strong>evaluate its impact</strong> on income?</td>
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<td>- <strong>Average treatment effect</strong> (ATE) is not informative</td>
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<td>- <strong>Effect of treatment on the treated</strong> (ETT) focuses on motivated individuals who participate</td>
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Motivation continued

Epidemiology

- A drug is administered by a doctor to patients she feels will benefit
- The doctor’s hunch is a confounder for the effect of treatment
- ATE cannot be identified unless additional assumptions are made
- ETT offers a substitute measure of effectiveness
Counterfactual notation

- $T$ is binary treatment, $Y$ is response
- $Y_t(u)$ is the response of unit $u$ to treatment $T = t$
- If $u$ receives $T = 1$, $Y_0(u)$ is counterfactual

Counterfactual Definition

$$ETT = E(Y_1 - Y_0 | T = 1, X)$$

Heckman and Robb (1985)
Problems with ETT

Well-defined?
- The ETT as defined above apparently depends on joint distribution of \((Y_1, Y_0, T)\)
- Is ETT well defined?

Identifiable?
- ETT involves term \(E(Y_0|T = 1)\)
- Observational regimes - no intervention
- Possible approaches: instrumental variables and control functions
Decision theoretic (DT) set-up

Motivation

- No counterfactuals
- Want to answer
  - “which decision is best"
  - not
  - “what would have happened if ..."

Notation

- \( F_T \) intervention variable, \( Z \) other variables
- \( p(T = t | F_T = t, Z) = 1 \) means set \( T = t \)
- \( p(T | F_T = \emptyset, Z) = p(T | Z) \), \( T \) arises "naturally"
DT continued

Variables
- $T$ treatment, $Y$ response
- $U$ - motivation (econ), doc’s hunch (epi), unobserved

Conditional independences
- $U \perp F_T$
- $Y \perp F_T|(U, T)$
ETT in DT continued

**DT Definition**

\[
ATE(U) = \mathbb{E}\{E(Y|FT = 1, U) - E(Y|FT = 0, U) | T = 1\}
\]

**Well-defined?**

- Apparently depends on \( U \) - well defined?
- By manipulating conditional independences

\[
ETT = \frac{E(Y|FT = \emptyset) - E(Y|FT = 0)}{p(T = 1|FT = \emptyset)}
\]

Note: different hospitals will have different \( U \)'s!
### Identifiability

#### Issues
- No controlled intervention
- NEED Additional data OR
- additional assumptions

#### Approaches
- IV methods
- Reformulate and use control functions
- Rubin (1974) uses method of matching
Instrumental variable method

Definition of IV

A variable $A$ is an instrument for $T$ if

- $A \not\perp T$
- $A \perp (F_T, U)$
- $Y \perp (A, F_T) \mid (U, T)$

$A$ can be used in place of $F_T$
IV continued

\[ ETT = \frac{\left\{ \begin{array}{l} (a) \quad E(Y|F_T = \emptyset) \\ (b) \quad E(Y|F_T = 0) \end{array} \right\}}{p(T = 1|F_T = \emptyset)} \]

**Example**

- Doctor decides who to treat \((F_T = \emptyset)\)
- can estimate \((a)\) and \((c)\) from observation
- But \((b) = E(Y|F_T = 0)\) presents a problem
Example continued

- Some patients are allergic to drug
- so they cannot be treated!
- Use allergy $A$ as the instrument
- so $E(Y|A = 1) \equiv E(Y|F_T = 0)$
- $(b)$ is the response of the allergic patients
Conclusions

- ETT useful when intervention not possible
  OR
- when ATE does not tell whole story
- ETT well-defined (even in counterfactual terms)
- ETT can be identified using
  - IV - good as only control group needed
  - BUT - need “forcing instrument”!
  - Control functions


Didelez, V. and Sheehan, N. Mendelian randomisation and instrumental variables: What can and what can’t be done. Research Report 05-02, Department of Health Sciences, University of Leicester, 2005