The effect of treatment on the treated: a decision theoretic perspective

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Outline

- Motivation
- Counterfactual definition
- Problems
- Intro to Decision theoretic approach (DT)
- DT definition
- Identifiability
- Intrumental variables
- Conclusions

Motivation

Econometrics

- A programme for adult learning is opened in a community
- How do you evaluate its impact on income?
- Average treatment effect (ATE) is not informative
- Effect of treatement on the treated (ETT) focuses on motivated individuals who participate

Motivation continued

Epidemiology

- A drug is administered by a doctor to patients she feels will benefit
- The doctor's hunch is a confounder for the effect of treatment
- ATE cannot be identified unless additional assumptions are made
- ETT offers a substitute measure of effectiveness

Econometric definition of ETT

Counterfactual notation

- T is binary treatment, Y is response
- $Y_t(u)$ is the response of unit u to treatment $T = t$
- If u recieves $T = 1$, $Y_0(u)$ is counterfactual

Counterfactual Definition

$$ETT = E(Y_1 - Y_0 | T = 1, X)$$

Heckman and Robb (1985)

Problems with ETT

Well-defined?

- The ETT as defined above apparently depends on joint distribution of (Y_1, Y_0, T)
- Is ETT well defined?

Identifiable?

- ETT involves term $E(Y_0|T = 1)$
- Observational regimes - no intervention
- Possible approaches: instrumental variables and control functions

Decision theoretic (DT) set-up

Motivation

- No counterfactuals
- Want to answer
  - “which decision is best"
  - not
  - “what would have happened if ..."

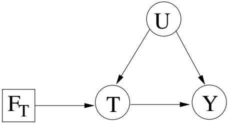
Notation

- F_T intervention variable, Z other variables
- $p(T = t | F_T = t, Z) = 1$ means set $T = t$
- $p(T | F_T = \emptyset, Z) = p(T | Z)$, T arises “naturally"

DT continued

Variables

- T treatment, Y response
- U - motivation (econ), doc's hunch (epi)
  unobserved



Conditional independences

- $U \perp\!\!\!\perp F_T$
- $Y \perp\!\!\!\perp F_T | (U, T)$

ETT in DT continued

DT Definition

$$E\{\overbrace{E(Y|F_T = 1, {\color{red}U}) - E(Y|F_T = 0, {\color{red}U})}^{ATE(U)}|T = 1\}$$

Well-defined?

- Apparently depends on U - well defined?
- By manipulating conditional independences

$$ETT = \frac{E(Y|F_T = \emptyset) - E(Y|F_T = 0)}{p(T = 1|F_T = \emptyset)}$$

Note: different hospitals will have different U's!

Identifiability

Issues

- No controlled intervention
- NEED Additional data OR
- additional assumptions

Approaches

- IV methods
- Reformulate and use control functions
- Rubin (1974) uses method of matching

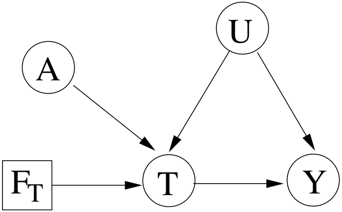
Instrumental variable method

Definition of IV

A variable A is an instrument for T if

- $A \not\perp\!\!\!\perp T$
- $A \perp\!\!\!\perp (F_T, U)$
- $Y \perp\!\!\!\perp (A, F_T) | (U, T)$

A can be used in place of F_T



IV continued

$$ETT = \frac{\overbrace{E(Y|F_T = \emptyset)}^{(a)} - \overbrace{E(Y|F_T = 0)}^{(b)}}{\underbrace{p(T = 1|F_T = \emptyset)}_{(c)}}$$

Example

- Doctor decides who to treat ($F_T = \emptyset$)
- can estimate (a) and (c) from observation
- But (b)$= E(Y|F_T = 0)$ presents a problem

IV continued

$$ETT = \frac{\overbrace{E(Y|F_T = \emptyset)}^{(a)} - \overbrace{E(Y|F_T = 0)}^{(b)}}{\underbrace{p(T = 1|F_T = \emptyset)}_{(c)}}$$

Example continued

- Some patients are allergic to drug
- so they cannot be treated!
- Use allergy A as the instrument
- so $E(Y|A = 1) \equiv E(Y|F_T = 0)$
- (b) is the response of the allergic patients

Conclusions

- ETT useful when intervention not possible OR
- when ATE does not tell whole story
- ETT well-defined (even in counterfactual terms)
- ETT can be identified using
  - IV - good as only control group needed
  - BUT - need "forcing instrument"!
  - Control functions

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