What are graphical models?

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Outline

Introduction

Uses

- Physics
- Genetics
- Psychology - Path analysis, Structural equation models
- Statistics
- Causal inference

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- Physics
- Genetics
- Psychology - Path analysis, Structural equation models
- Statistics
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Types

- Directed
- Directed Acyclic
- Unidirected
- Chain graphs

What is a DAG?

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- All arrows have direction

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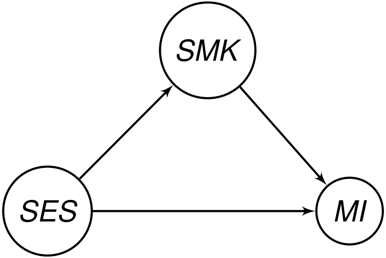

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DAGs are used to encode conditional independence statements

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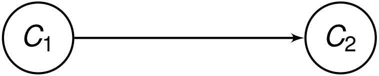
- which means $p(A, C|B) = p(A|B)p(C|B)$
- Although DAGs have arrows, they DO NOT automatically mean causal relationships
- rather an arrow means dependence/association and lack of an arrow means independence/no association

Simple example - inheritance



1. Two children are siblings
2. If you know the DNA of one, you know something about the DNA of the other
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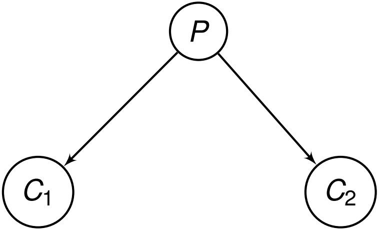

1. Two children are siblings
2. If you know the DNA of one, you know something about the DNA of the other
3. they are associated
4. If you know their parents' DNA however
5. knowing about one child tells you nothing new about the other
6. they are independent GIVEN the parents

Qualitative approach

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- a DAG that expresses assumptions about relationships (i.e. pre-data analysis) does not necessarily correspond to reality

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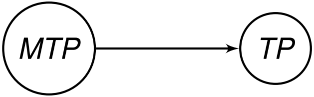
Caveats

- a DAG that expresses assumptions about relationships (i.e. pre-data analysis) does not necessarily correspond to reality
- Putative associations/causal relations need to be tested against data where possible and assessed carefully

Constructing a DAG



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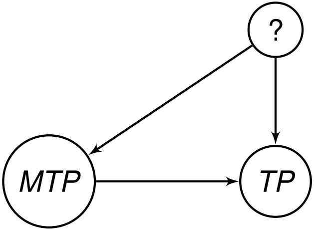

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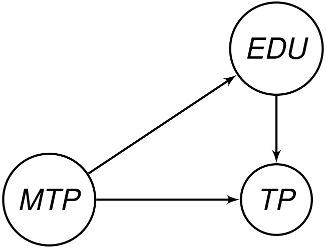

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- But surely that is influenced in its own way by?? Anyone?

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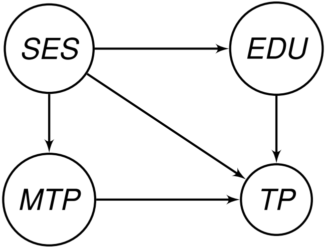

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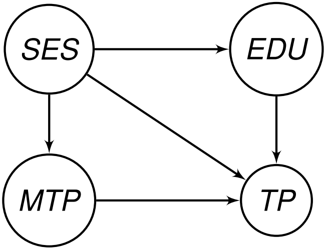
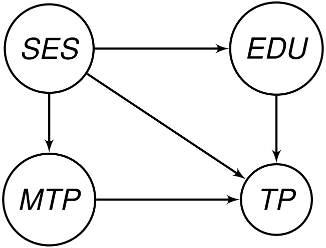


- This is a simple example - could add
  - Ethnicity

Constructing a DAG



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  - Low-self esteem

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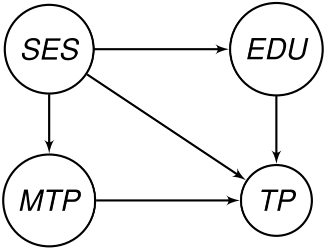

- This is a simple example - could add
  - Ethnicity
  - Low-self esteem
  - Substance abuse

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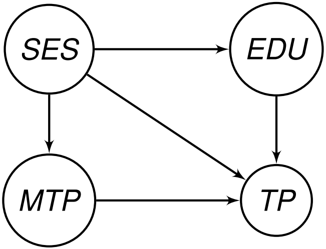

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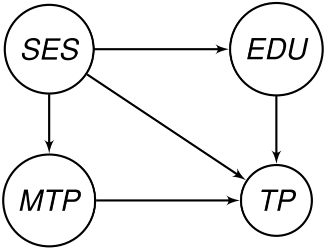

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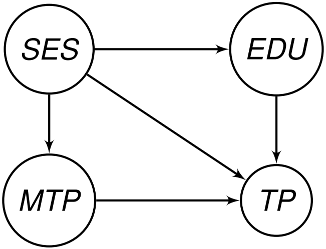

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  - Ethnicity
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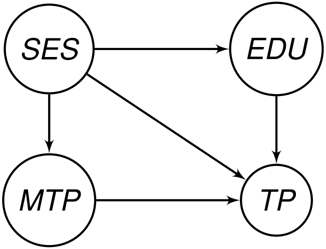
- Some of these could be unobserved or reported with bias
- e.g. low-self esteem or substance abuse

Incorporating data



$$Pr(TP|MTP) = \sum_{SES,EDU} Pr(TP|MTP,SES,EDU)Pr(EDU|SES)Pr(MTP|SES)Pr(SES)$$

can use frequencies from contingency tables to estimate $Pr(TP|MTP)$ and Odds Ratio

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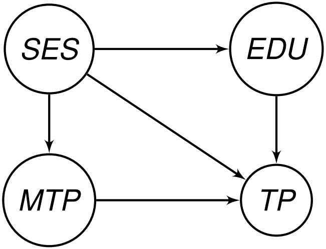

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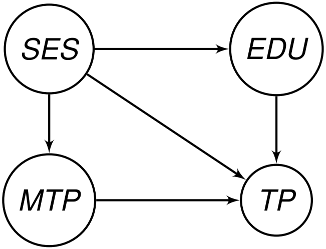

$$
\begin{aligned}
Pr(TP|MTP) &= \\
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\end{aligned}
$$

can use frequencies from contingency tables to estimate $Pr(TP|MTP)$ and Odds Ratio

- The graph tells us how to factorise the distribution of variables into smaller simple parts
- Helps to estimate using a modular approach - see later

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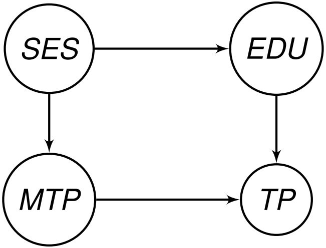

- We can do a path analysis [2] by assuming linear relationships between the variables
- For example, if we think that the influence of SES on TP is mediated only by MTP and EDU
- i.e. $TP \perp\!\!\!\perp SES | (MTP, EDU)$ then

Incorporating data

$$
\begin{tikzcd}
SES \arrow[r, "\delta"] \arrow[d, "\gamma"] & EDU \arrow[d, "\beta"] \\
MTP \arrow[r, "\alpha"] & TP
\end{tikzcd}
$$

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- For example, if we think that the influence of SES on TP is mediated only by MTP and EDU
- i.e. $TP \perp\!\!\!\perp SES | (MTP, EDU)$ then
- $TP = \mu_1 + \alpha MTP + \beta EDU + \epsilon$
- $MTP = \mu_2 + \gamma SES + \epsilon$ and $EDU = \mu_3 + \delta SES + \epsilon$

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True?

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2. Moralisation criteria (see next slide)
3. Use e.g. χ^2 or Mantel-Haenszel test (or Bayesian network software) to determine if hold in data

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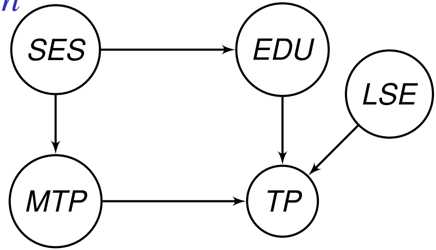
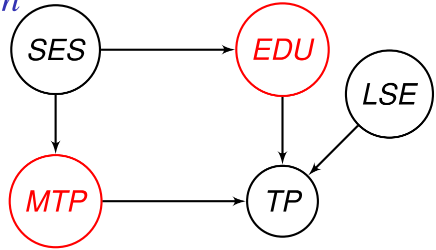
1. What conditional independences does DAG encode?
2. Moralisation criteria (see next slide)
3. Use e.g. χ^2 or Mantel-Haenszel test (or Bayesian network software) to determine if hold in data
4. Regressions - if adding a variable to reg makes no difference to the outcome - maybe there is no dependence (not 100%).

Moralisation



- Say you care about relationship between EDU and MTP

Moralisation



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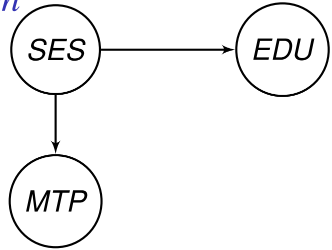

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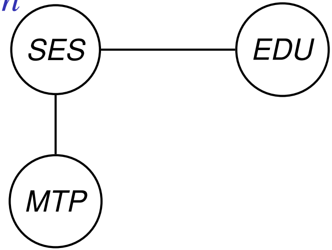

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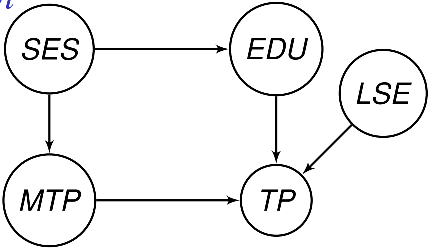
- all paths from EDU and MTP go through SES - $MTP \perp\!\!\!\perp EDU | SES$
- i.e. mother being a teen mum is only associated to daughter's education via SES - makes sense?

Moralisation



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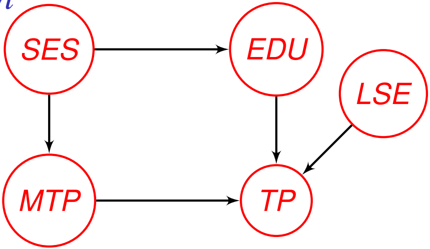

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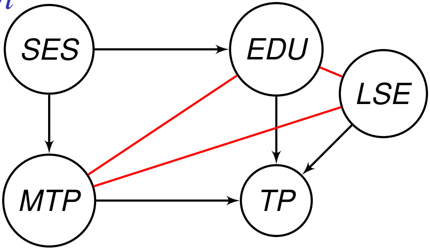

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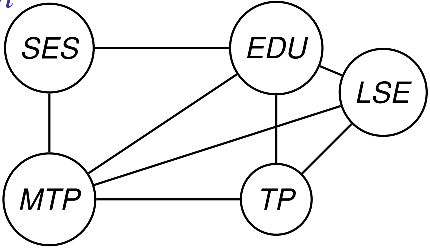

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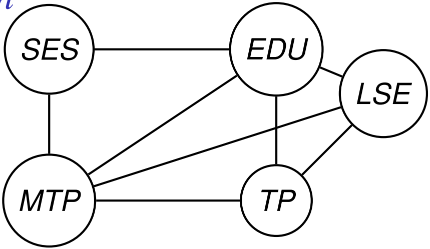

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- Join parents of common children
- remove direction from arrows
- all paths from SES and TP go through EDU and MTP - $TP \perp\!\!\!\perp SES | (MTP, EDU)$
- i.e. being a teen mum is only associated to SES via mother's teen mum status and education - not plausible, need more confounders!

Getting DAGs from data

Data mining

- There are various methods for extracting DAGs from data
- Most ask what the conditional independences are between variables (using e.g. χ^2 tests) and construct a series of DAGs

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- There are various methods for extracting DAGs from data
- Most ask what the conditional independences are between variables (using e.g. χ^2 tests) and construct a series of DAGs
- There are also loads of computer programmes that take data and turn it into DAGs

Simple example

Political affiliation (PA), abuse as a child (AC) and abusive parent (AP) [3]

Contingency table

Obs		PA			
AC	AP	l	s	r	tot
1	1	**12**	**27**	**58**	
	0	**7**	**28**	**30**	
0	1	**9**	**5**	**9**	
	0	**19**	**15**	**18**	
	tot				

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		19	55	88	162			19	55	88	162
0	1	**9**	**5**	**9**	23	0	1	**9**	**6**	**8**	23
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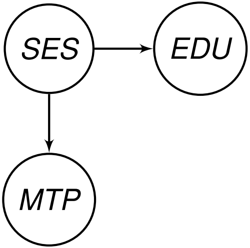
	0	**19**	**15**	**18**	52		0	**19**	**14**	**19**	52
	tot	28	20	27	75		tot	28	20	27	75

The two tables are very similar and "say" that $PA \perp\!\!\!\perp AP | AC$

DAGs are modular



- Data source 1: SES,EDU, MTP

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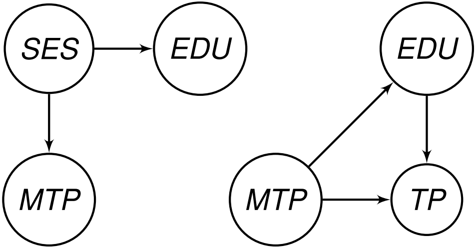

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- Data source 2: MTP, EDU and TP

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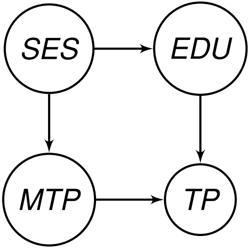

- Data source 1: SES,EDU, MTP
- Data source 2: MTP, EDU and TP
- Can join two sources to make inference about SES and TP!

Causal inference

Types

- Potential outcomes/Counterfactuals (Rubin [4],Pearl [5])
- Causal Graphs (Pearl[5],Greenland, Robins [6])
- Decision theory (Dawid [7],Geneletti [8], Didelez [9])

General issues

- no causation w/out manipulation
- Means need to be careful about observational data
- typically there are unobserved confounders, reporting bias etc
- Causality is an external assumption

Advert!

- One day course in “How to use graphical models to understand relationships between variable”
- Date TBA - April 2009
- email me s.geneletti@imperial.ac.uk for further info!

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